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Periodic permeable interface cracks in piezoelectric materials

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Abstract

This work revisits the generalized 2D problem of electrically permeable collinear interface cracks in piezoelectric materials with two motivations: one is to present a more explicit approach to the considered problem; the other is to derive some new results for periodical interface crack problem in piezoelectric materials with the use of the new approach. Based on the Stroh formalism, the mechanical–electric coupling boundary equations are decoupled into two equations: the first one is related only to the applied mechanical loads, and the second one only to the applied electric field. According to the traditional method or available results, the solution for the first equation can be given, and then the solution for the second equation can be directly written out by using the results of the first equation. Furthermore, the solutions for infinite number of periodical collinear interface cracks are at the first time presented in closed form. The solutions include the field intensity factors and the electric fields both inside and outside the cracks. It is shown that under the electric loading only, the electric fields are uniform not only in the materials but also inside the cracks, while the stress is zero wherever. However, when the combined mechanical–electric loadings are applied at infinity, the electric fields inside the cracks may be singular and oscillatory, and such is the case for the stresses near the crack tips, but the intensities of all singularities depend on the material properties and the applied mechanical loads, not on the applied electric loads. Finally, a numerical example is given for the case of a single interface crack, and the electric fields inside the crack is shown analytically and graphically.

Keywords: Piezoelectric materials; Electrically permeable; Periodical interface cracks

1. Introduction

Fracture mechanics of piezoelectric materials has received considerable interest in the recent decade. A lengthy literature has been presented in recent review papers (McMeeking, 1999; Kamlah, 2001; Zhang et al., 2002; and Zhang and Gao, 2003). It can be found that the interface crack problems in piezoelectric media have been well studied. Main contributions to the generalized two-dimensional problem of interface cracks between two dissimilar piezoelectric half-spaces include the works of Kuo and Barnett (1991), Suo et al. (1992), Liang and Hwu (1996), Beom and Atluri (1996), Qin and Mai (1999), Ma and Chen (2001),

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Herrmann et al. (2001), Herrmann and Loboda (2003a,b) for an impermeable crack; Govorukha and Loboda (2000), Ru (2000) and Beom and Atluri (2002) for a conducting crack; and also Wang and Han (1999), Gao and Wang (2000), Herrmann and Loboda (2000, 2003a,b), and Liu and Hsia (2003) for a permeable crack. Recently, Wang and Shen (2002) give a general treatment on various interface defects at anisotropic piezoelectric bimaterial interface.

However, to the authors' knowledge, no solutions are presented for the case of periodic interfacial cracks in two dissimilar piezoelectric materials, though the similar problem has been solved for the case of a homogeneous piezoelectric medium (Gao and Wang, 1999; Hao, 2001). This work revisits the generalized two-dimensional problem of interfacial cracks in piezoelectric materials with two motivations: one is to develop a more concise and explicit approach to the general collinear cracks, with which one can reduce a piezoelectric interface crack problem to an equivalent one to that in purely elastic anisotropic media; the other is to derive the solution for infinite number of periodical collinear interface cracks. Since a crack in piezoelectric solids behaves more like a permeable slit within the scope of linear elasticity (Shindo et al., 2002), the permeable crack model is used in the present work, and special attention is played to the examination of the electric fields within the crack.

Below is the plan of this work: following the brief introduction, Section 2 outlines the Stroh formalism. In Section 3 the general solutions for N collinear permeable interface cracks are derived. Furthermore, the solutions for infinite number of collinear periodic cracks are presented in Section 4. As a special example, analytical solutions and numerical results are given for a single interface crack, and especially the electric field inside the crack is calculated and shown graphically in Section 5. Finally, Section 6 concludes the work.

2. Basic equations

In a rectangular coordinate system x_i $(i = 1, 2, 3)$, the basic equations for a linear piezoelectric solid are (Barnett and Lothe, 1975)

$$\sigma_{ij,j} = 0, \quad D_{i,i} = 0, \tag{1}$$

$$\gamma_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\varphi_{,i}, \tag{2}$$

$$\sigma_{ij} = c_{ijkl}\gamma_{kl} - e_{kij}E_k, \quad D_k = e_{kij}\gamma_{ij} + \varepsilon_{kl}E_l, \tag{3}$$

where u_i, φ, σ_{ij}, γ_{ij}, D_j and E_i are the displacement, the electric potential, the stress, the strain, the electric displacement and the electric field, respectively, and c_{ijkl}, e_{ijk} and ε_{ij} stand for the elastic constants, the piezoelectric constants and the dielectric constants, respectively.

Consider a generalized two-dimensional problem in which all the field variables are independent of x_3. We introduce a generalized displacement vector $\mathbf{u}$ as (Barnett and Lothe, 1975)

$$\mathbf{u} = [u_1, u_2, u_3, \varphi]^{\mathrm{T}} = \mathbf{a}f(x_1 + px_2), \tag{4}$$

where the superscript 'T' represents the transpose, $f(x_1 + px_2)$ is an analytic function, p is a complex number, and $\mathbf{a}$ a constant four-element column vector. Eqs. (1)–(3) can be satisfied by (4) for arbitrary $f(x_1 + px_2)$ if

$$[\mathbf{W} + p(\mathbf{R} + \mathbf{R}^{\mathrm{T}}) + p^2\mathbf{T}]\mathbf{a} = \mathbf{0}, \tag{5}$$

where the matrices **Q**, **R** and **T** are given by

$$\mathbf{W} = \begin{bmatrix} c_{i1k1} & e_{11i} \\ e_{11i}^{\mathrm{T}} & -\varepsilon_{11} \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} c_{i1k2} & e_{21i} \\ e_{12i}^{\mathrm{T}} & -\varepsilon_{12} \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} c_{i2k2} & e_{22i} \\ e_{22i}^{\mathrm{T}} & -\varepsilon_{22} \end{bmatrix}, \quad i,k = 1,2,3. \tag{6}$$

The condition for non-trivial solutions of (5) requires

$$|\mathbf{W} + p(\mathbf{R} + \mathbf{R}^{\mathrm{T}}) + p^2\mathbf{T}| = 0. \tag{7}$$

It can be shown that Eq. (7) has eight roots p_α and $\bar{p}_\alpha$ ($\alpha = 1,2,3,4$) where p_α cannot be real because of the positive definiteness of the strain energy and electric energy densities. It is convenient to calculate p_α by solving the following standard eigen-equation:

$$\mathbf{N}\zeta = p\zeta,$$

where

$$\mathbf{N} = \begin{pmatrix} \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{N}_3 & \mathbf{N}_1^{\mathrm{T}} \end{pmatrix}, \quad \zeta = \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix},$$

$$\mathbf{N}_1 = -\mathbf{T}^{-1}\mathbf{R}^{\mathrm{T}}, \quad \mathbf{N}_2 = \mathbf{T}^{-1} = \mathbf{N}_2^{\mathrm{T}}, \quad \mathbf{N}_3 = \mathbf{R}\mathbf{T}^{-1}\mathbf{R}^{\mathrm{T}} - \mathbf{W} = \mathbf{N}_3^{\mathrm{T}}.$$

The general solution of Eqs. (1)–(3) can be expressed as

$$\mathbf{u} = \mathbf{A}\mathbf{f}(z) + \overline{\mathbf{A}}\overline{\mathbf{f}(z)}, \tag{8}$$

$$\boldsymbol{\phi} = \mathbf{B}\mathbf{f}(z) + \overline{\mathbf{B}}\overline{\mathbf{f}(z)}, \tag{9}$$

where

$$\mathbf{A} = (\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4), \quad \mathbf{B} = (\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4),$$

$$\mathbf{f}(z_\alpha) = [f_1(z_1), f_2(z_2), f_3(z_3), f_4(z_4)]^{\mathrm{T}}, \quad z_\alpha = x_1 + p_\alpha x_2$$

and $\boldsymbol{\phi}$ is the generalized stress function such that

$$\boldsymbol{\sigma}_2 = [\sigma_{2j}, D_2]^{\mathrm{T}} = \boldsymbol{\phi}_{,1}, \quad \boldsymbol{\sigma}_1 = [\sigma_{1j}, D_1]^{\mathrm{T}} = -\boldsymbol{\phi}_{,2}. \tag{10}$$

In addition, the **A** and **B** have the following nature (Ting, 1996)

$$\begin{bmatrix} \mathbf{B}^{\mathrm{T}} & \mathbf{A}^{\mathrm{T}} \\ \overline{\mathbf{B}}^{\mathrm{T}} & \overline{\mathbf{A}}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{A} & \overline{\mathbf{A}} \\ \mathbf{B} & \overline{\mathbf{B}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \tag{11}$$

where **I** is a 4×4 unit matrix.

3. Solution of finite collinear cracks

Consider finite number of collinear cracks l_n between two half-infinite piezoelectric planes s_1 and s_2, as shown in Fig. 1. The union of the cracks and uncracked part in the x_1-axis are denoted by L_c and L_b, respectively. Assume that these two half-planes coexist in the state of generalized two-dimensional deformation under piecewise uniform loads at infinity (Gao and Wang, 2000). Additionally, the cracks are assumed to be traction-free, and electrically permeable slits. In this case the boundary conditions can be expressed as (Parton, 1976)

$$\sigma_{2j}^+ = \sigma_{2j}^- = 0, \quad D_2^+ = D_2^-, \quad E_1^+ = E_1^-, \quad \text{on } L_c, \tag{12}$$

$$u_j^+ = u_j^-, \quad D_2^+ = D_2^-, \quad E_1^+ = E_1^-, \quad \text{on } L_b. \tag{13}$$

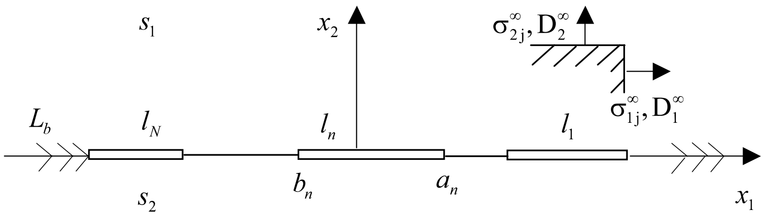

Fig. 1. N collinear interface cracks in piezoelectric bimaterials.

From (8) and (9) one has

$$\mathbf{u}_{,1} = \mathbf{A}\mathbf{F}(z) + \overline{\mathbf{A}}\,\overline{\mathbf{F}(z)}, \tag{14}$$

$$\phi_{,1} = \mathbf{B}\mathbf{F}(z) + \overline{\mathbf{B}}\,\overline{\mathbf{F}(z)}, \tag{15}$$

where $\mathbf{F}(z) = \mathrm{d}\mathbf{f}/\mathrm{d}z$.

For the present problem, $\mathbf{F}(z)$ has the form of

$$\mathbf{F}_k(z) = \mathbf{c}_k^{\infty} + \mathbf{F}_{k0}(z), \quad k = 1, 2, \tag{16}$$

where $\mathbf{c}_k^{\infty}$ is a constant vector; $\mathbf{F}_{k0}(z)$ is a unknown function vector in s_1 $(k = 1)$ or in s_2 $(k = 2)$, and $\mathbf{F}_{k0}(\infty) = \mathbf{0}$.

First let us determine $\mathbf{c}_k^{\infty}$. It is obvious that $\mathbf{c}_k^{\infty}$ is the complex potential corresponding to two completely bonded half-planes subjected to the applied uniform loads at infinity. For the subproblem, the continuous conditions of deformation and stress on the entire x_1-axis, from (12) and (13), require

$$\begin{aligned} \mathbf{A}_1\mathbf{c}_1^{\infty} + \overline{\mathbf{A}}_1\bar{\mathbf{c}}_1^{\infty} &= \mathbf{A}_2\mathbf{c}_2^{\infty} + \overline{\mathbf{A}}_2\bar{\mathbf{c}}_2^{\infty} = \mathbf{u}_{,1}^{\infty}, \\ \mathbf{B}_1\mathbf{c}_1^{\infty} + \overline{\mathbf{B}}_1\bar{\mathbf{c}}_1^{\infty} &= \mathbf{B}_2\mathbf{c}_2^{\infty} + \overline{\mathbf{B}}_2\bar{\mathbf{c}}_2^{\infty} = \boldsymbol{\phi}_{,1}^{\infty}, \end{aligned} \tag{17}$$

where

$$\boldsymbol{\phi}_{,1}^{\infty} = [\sigma_{21}^{\infty}, \sigma_{22}^{\infty}, \sigma_{23}^{\infty}, D_2^{\infty}]^{\mathrm{T}}, \quad \mathbf{u}_{,1}^{\infty} = [u_{1,1}^{\infty}, u_{2,1}^{\infty}, u_{3,1}^{\infty}, u_{4,1}^{\infty}]^{\mathrm{T}}$$

in which $\boldsymbol{\phi}_{,1}^{\infty}$ is the given loading at infinity, $\mathbf{u}_{,1}^{\infty}$ is related to the generalized strain at infinity, and it can be expressed by the generalized stress at infinity from using the constitutive equations.

Noting (11), one has from (17) that

$$\mathbf{c}_k^{\infty} = \mathbf{B}_k^{\mathrm{T}}\mathbf{u}_{,1}^{\infty} + \mathbf{A}_k^{\mathrm{T}}\phi_{,1}^{\infty}, \quad (k = 1, 2). \tag{18}$$

The remainder task is to find $\mathbf{F}_{k0}(z)$ in (16). Using (12) and (13) one has

$$\phi_{,1}(x_1^+) = \phi_{,1}(x_1^-), \quad -\infty < x_1 < \infty. \tag{19}$$

Substituting (15) together with (16) into (19), and then using (17) results in

$$\mathbf{B}_1\mathbf{F}_{10}(x_1^+) + \overline{\mathbf{B}}_1\overline{\mathbf{F}}_{10}(x_1^-) = \mathbf{B}_2\mathbf{F}_{20}(x_1^-) + \overline{\mathbf{B}}_2\overline{\mathbf{F}}_{20}(x_1^+), \quad -\infty < x_1 < \infty. \tag{20}$$

From (20) one obtains (Muskhelishvili, 1975)

$$\begin{aligned} \mathbf{B}_1\mathbf{F}_{10}(z) - \overline{\mathbf{B}}_2\overline{\mathbf{F}}_{20}(z) &= \mathbf{0}, \quad z \in s_1, \\ \mathbf{B}_2\mathbf{F}_{20}(z) - \overline{\mathbf{B}}_1\overline{\mathbf{F}}_{10}(z) &= \mathbf{0}, \quad z \in s_2. \end{aligned} \tag{21}$$

Introduce a jump function

$$\mathrm{i}\Delta\mathbf{u}_{,1} = \mathrm{i}[\mathbf{u}_{,1}(x_1^+) - \mathbf{u}_{,1}(x_1^-)]. \tag{22}$$

Using (14), (16), (17) and (21), Eq. (22) can be reduced to

$$\mathrm{i}\Delta\mathbf{u}_{,1} = \mathbf{K}^{+}(x_1) - \mathbf{K}^{-}(x_1), \tag{23}$$

where

$$\mathbf{K}(z) = \begin{cases} \mathbf{H}\mathbf{B}_1\mathbf{F}_{10}(z), & z \in s_1, \\ \overline{\mathbf{H}}\mathbf{B}_2\mathbf{F}_{20}(z), & z \in s_2, \end{cases} \tag{24}$$

$$\mathbf{H} = \mathbf{Y}_1 + \overline{\mathbf{Y}}_2, \quad \mathbf{Y}_k = \mathrm{i}\mathbf{A}_k\mathbf{B}_k^{-1}.$$

Using the third equations of (12) and (13) we have from (23) that

$$K_4^{+}(x_1) - K_4^{-}(x_1) = 0, \quad -\infty < x_1 < \infty. \tag{25}$$

Noting $\mathbf{K}(\infty) = \mathbf{0}$ from (24), the solution of (25) is (Muskhelishvili, 1975)

$$K_4(z) = 0. \tag{26}$$

On the other hand, substituting (16) into (15) results in

$$\boldsymbol{\phi}_{,1} = \boldsymbol{\phi}_{,1}^{\infty} + \mathbf{B}_1\mathbf{F}_{10}(x_1) + \overline{\mathbf{B}}_1\overline{\mathbf{F}_{10}(x_1)}. \tag{27}$$

Using (21), (27) becomes

$$\boldsymbol{\phi}_{,1} = \boldsymbol{\phi}_{,1}^{\infty} + \mathbf{B}_1\mathbf{F}_{10}(x_1) + \mathbf{B}_2\mathbf{F}_{20}(x_1). \tag{28}$$

Considering (24), one can rewrite (28) as

$$\boldsymbol{\phi}_{,1} = \boldsymbol{\phi}_{,1}^{\infty} + \boldsymbol{\Lambda}\mathbf{K}^{+}(x_1) + \overline{\boldsymbol{\Lambda}}\mathbf{K}^{-}(x_1), \tag{29}$$

where

$$\boldsymbol{\Lambda} = \mathbf{H}^{-1} = \begin{bmatrix} \boldsymbol{\Lambda}_{\sigma} & \boldsymbol{\Lambda}_{3\times 1}, \\ \boldsymbol{\Lambda}_{1\times 3} & \boldsymbol{\Lambda}_{44} \end{bmatrix}, \tag{30}$$

in which $\boldsymbol{\Lambda}_{\sigma}$ is a 3×3 upper left-hand block; $\boldsymbol{\Lambda}_{44}$ is a real element; $\boldsymbol{\Lambda}_{1\times 3}$ and $\boldsymbol{\Lambda}_{3\times 1}$ are a row and column vector, respectively. It can be shown that $\boldsymbol{\Lambda}_{\sigma}$ is positive definite (Suo et al., 1992).

Taking the first three rows and the fourth row of (29), respectively, and then using (26) yields

$$\boldsymbol{\sigma}_2(x_1) = \boldsymbol{\sigma}_2^{\infty} + \boldsymbol{\Lambda}_{\sigma}\mathbf{K}_3^{+}(x_1) + \overline{\boldsymbol{\Lambda}_{\sigma}}\mathbf{K}_3^{-}(x_1), \tag{31}$$

$$D_2(x_1) = D_2^{\infty} + \boldsymbol{\Lambda}_{1\times 3}\mathbf{K}_3^{+}(x_1) + \overline{\boldsymbol{\Lambda}_{1\times 3}}\mathbf{K}_3^{-}(x_1), \tag{32}$$

where

$$\mathbf{K}_3 = [K_1, K_2, K_3]^{\mathrm{T}}, \quad \boldsymbol{\sigma}_2^{\infty} = [\sigma_{21}^{\infty}, \sigma_{22}^{\infty}, \sigma_{23}^{\infty}]^{\mathrm{T}}.$$

Eq. (32) can be rewritten as

$$D_2(x_1) = D_2^{\infty} + [\boldsymbol{\Lambda}_{1\times 3}\boldsymbol{\Lambda}_{\sigma}^{-1}]\boldsymbol{\Lambda}_{\sigma}\mathbf{K}_3^{+}(x_1) + [\overline{\boldsymbol{\Lambda}}_{1\times 3}\overline{\boldsymbol{\Lambda}}_{\sigma}^{-1}]\overline{\boldsymbol{\Lambda}}_{\sigma}\mathbf{K}_3^{-}(x_1). \tag{33}$$

On the crack faces, $\boldsymbol{\sigma}_2 = \mathbf{0}$, and thus (31) gives

$$\boldsymbol{\Lambda}_{\sigma}\mathbf{K}_3^{+}(x_1) + \overline{\boldsymbol{\Lambda}_{\sigma}}\mathbf{K}_3^{-}(x_1) = -\boldsymbol{\sigma}_2^{\infty}, \quad x_1 \in L_{\mathrm{c}}, \tag{34}$$

that is

$$\overline{\boldsymbol{\Lambda}_{\sigma}}\mathbf{K}_3^{-}(x_1) = -\boldsymbol{\sigma}_2^{\infty} - \boldsymbol{\Lambda}_{\sigma}\mathbf{K}_3^{+}(x_1), \quad x_1 \in L_{\mathrm{c}}. \tag{35}$$

Substituting (35) into (33) yields the electric displacement on crack surface as

$$D_2^0(x_1) = D_2^\infty - [\overline{\mathbf{\Lambda}}_{1\times3}\overline{\mathbf{\Lambda}}_\sigma^{-1}]\boldsymbol{\sigma}_2^\infty + [\mathbf{\Lambda}_{1\times3}\mathbf{\Lambda}_\sigma^{-1} - \overline{\mathbf{\Lambda}}_{1\times3}\overline{\mathbf{\Lambda}}_\sigma^{-1}]\mathbf{\Lambda}_\sigma\mathbf{K}_3^+(x_1), \quad x_1 \in L_c. \tag{36}$$

Up to here one finds that the mechanical–electrical boundary condition (29) is decoupled into two equations: one is (33) and the other is (34). Eq. (34) is related only to the applied mechanical load $\boldsymbol{\sigma}_2^\infty$. This means that our problem has been reduced to an equivalent interface crack problem to that in purely elastic anisotropic media, and thus the solution of $\mathbf{K}_3(z)$ can be obtained from (34) in terms of well-established method. Once $\mathbf{K}_3(z)$ is available, all mechanical and electric variables can be determined. It should be noted that the complete solution of $\mathbf{K}_3(z)$ needs the use of the single-valued condition of displacements such as

$$\int_{L_c} \Delta\mathbf{u}_{,1}\,\mathrm{d}x_1 = \mathbf{0}. \tag{37}$$

Substituting (23) into (37), and using (26) leads to

$$\int_{L_c} [\mathbf{K}_3^+(x_1) - \mathbf{K}_3^-(x_1)]\,\mathrm{d}x_1 = \mathbf{0}. \tag{38}$$

It can found from (34) and (38) that $\mathbf{K}_3(z)$ is not related to the applied electric loading. Thus, we here can conclude that the uniform electric loading at infinity has no influence on the fracture of an infinite piezoelectric solid with mathematical interface cracks.

When $\mathbf{H}$ is real, $\mathbf{\Lambda}$ is real too. Then, (34) and (36) become

$$[\mathbf{\Lambda}_\sigma\mathbf{K}_3(x_1)]^+ + [\mathbf{\Lambda}_\sigma\mathbf{K}_3(x_1)]^- = -\boldsymbol{\sigma}_2^\infty, \tag{39}$$

$$D_2^0(x_1) = D_2^\infty - \mathbf{\Lambda}_{1\times3}\mathbf{\Lambda}_\sigma^{-1}\boldsymbol{\sigma}_2^\infty. \tag{40}$$

The general solution of (39) can be easily written out. It can be shown that all fields behave the reverse square root singularities, and the stress intensity factor vector $\mathbf{k}_\sigma$ is the same as that of isotropic materials, while the intensity factor of electric displacement is $k_D = \mathbf{\Lambda}_{1\times3}\mathbf{\Lambda}_\sigma^{-1}\mathbf{k}_\sigma$. In addition, the electric field inside any crack is a constant, as shown in (40).

When $\mathbf{H}$ is complex and the electric loading is applied solely, we have from (34) that $\mathbf{K}_3(z) = \mathbf{0}$. With it, Eq. (36) leads to $D_2^0 = D_2^\infty$. This indicates that the stress is zero everywhere. The electric fields in the materials are equal to the applied ones, while inside the cracks one has $E_2^0 = D_2^\infty/\varepsilon_0 = E_2^\infty(\varepsilon_m/\varepsilon_0)$, where ε_m and ε_0 are the dielectric constants of the material and air, respectively. For most piezoelectric materials, $\varepsilon_m/\varepsilon_0$ is about 1000 and thus the electric field inside the cracks is about 1000 times higher than the applied electric field.

When $\mathbf{H}$ is complex but the combined mechanical–electric loadings are applied at infinity, $\mathbf{K}_3(z)$ has to be determined from (34). Upon obtaining $\mathbf{K}_3(z)$, the electric field $E_2^0 = D_2^\infty/\varepsilon_0$ inside the cracks can be given from (36). It is shown from (36) that E_2^0 may be singular and oscillatory near the tip of cracks, since $\mathbf{\Lambda}_{1\times3}\mathbf{\Lambda}_\sigma^{-1}$ is not real in general. Below we place our stress on this general case.

For this case, let

$$\mathbf{Q}^{-1}\mathbf{\Lambda}_\sigma^{-1}\overline{\mathbf{\Lambda}}_\sigma\mathbf{Q} = \langle\langle -\mathrm{e}^{-2\pi\mathrm{i}\delta_\alpha}\rangle\rangle, \tag{41}$$

where $\langle\langle\rangle\rangle$ indicates the diagonal matrix in which each component is varied according to the Greek index α, $\mathbf{Q}$ is the eigenvector matrix and δ_α is a complex number:

$$\delta_\alpha = -\frac{1}{2} + \mathrm{i}\varepsilon_\alpha.$$

The ε_α can be determined by (Ting, 1996)

$$\| -\mathrm{e}^{-2\pi\mathrm{i}\delta_\alpha}\mathbf{I} - \mathbf{\Lambda}_\sigma^{-1}\overline{\mathbf{\Lambda}}_\sigma \| = 0,$$

that is

$$\|\overline{\mathbf{\Lambda}}_\sigma - \mathrm{e}^{2\pi\varepsilon_\alpha}\mathbf{\Lambda}_\sigma\| = 0,$$

which has the solution in the form of (Ting, 1996)

$$\varepsilon_\alpha = (\varepsilon, -\varepsilon, 0), \tag{42}$$

where

$$\varepsilon = \frac{1}{\pi}\tan^{-1}\beta = \frac{1}{2\pi}\ln + \frac{1+\beta}{1-\beta}, \quad 0 \leqslant \beta = \left[-\frac{1}{2}\mathrm{tr}(\widehat{\mathbf{S}})^2\right]^{1/2}, \quad \widehat{\mathbf{S}} = \mathbf{D}^{-1}\mathbf{W}, \quad \mathbf{D} - \mathrm{i}\mathbf{W} = \overline{\Lambda}_\sigma.$$

Using (41), (34) can be decoupled and the general solution of $\mathbf{K}_3(z)$ can be obtained. Omitting some details we directly give the final result as

$$\mathbf{K}_3(z) = -\mathbf{Q}\left\langle\left\langle \frac{1 - X_\alpha(z)X_\alpha^{-1}(\infty)}{1 + \mathrm{e}^{2\pi\varepsilon_\alpha}} \right\rangle\right\rangle \mathbf{Q}^{-1}\mathbf{\Lambda}_\sigma^{-1}\sigma_2^\infty + \left\langle\left\langle \frac{X_\alpha(z)}{1 + \mathrm{e}^{2\pi\varepsilon_\alpha}} \right\rangle\right\rangle \mathbf{P}_{N-1}(z), \tag{43}$$

where

$$\mathbf{P}_{N-1}(z) = \mathbf{c}_{N-1}z^{N-1} + \cdots \mathbf{c}_0, \tag{44}$$

$$X_\alpha(z) = \prod_{n=1}^{N}(z - a_n)^{-1/2-\mathrm{i}\varepsilon_\alpha}(z - b_n)^{-1/2+\mathrm{i}\varepsilon_\alpha}, \tag{45}$$

$$X_\alpha^{-1}(\infty) = \prod_{n=1}^{N}\left[z - \frac{(a_n + b_n) + 2\mathrm{i}\varepsilon_\alpha(a_n - b_n)}{2}\right]. \tag{46}$$

The coefficients $\mathbf{c}_n$ involved in (44) can be determined by (38).

The stress intensity factor vector can be defined as (Wu, 1990)

$$\mathbf{k}_\sigma = \lim_{r\to 0}\sqrt{2\pi r}[\mathbf{\Lambda}_\sigma\mathbf{Q}]\langle\langle r^{-\mathrm{i}\varepsilon_\alpha}\rangle\rangle[\mathbf{\Lambda}_\sigma\mathbf{Q}]^{-1}\sigma_2^s(x_1), \tag{47}$$

where r means the distance from the crack tip; $\boldsymbol{\sigma}_2^s(x_1)$ stands for the singular principle part of generalized stress vector $\boldsymbol{\sigma}_2(x_1)$ ahead of the crack tip.

Ahead of the crack tips, it can be shown that

$$[\mathbf{\Lambda}_\sigma\mathbf{Q}]^{-1}\sigma_2^s(x_1) = \langle\langle X_\alpha(x_1)X_\alpha^{-1}(\infty)\rangle\rangle[\mathbf{\Lambda}_\sigma\mathbf{Q}]^{-1}\boldsymbol{\sigma}_2^\infty + \langle\langle X_\alpha(x_1)\rangle\rangle\mathbf{P}_{N-1}(x_1), \quad x_1 \in L_b. \tag{48}$$

Inserting (48) into (47) results in the general expression of stress intensity factors as

$$\mathbf{k}_\sigma = \lim_{r\to 0}\sqrt{2\pi r}[\mathbf{\Lambda}_\sigma\mathbf{Q}]\langle\langle r^{-\mathrm{i}\varepsilon_\alpha}\rangle\rangle[\langle\langle X_\alpha(x_1)X_\alpha^{-1}(\infty)\rangle\rangle[\mathbf{\Lambda}_\sigma\mathbf{Q}]^{-1}\boldsymbol{\sigma}_2^\infty + \langle\langle X_\alpha(x_1)\rangle\rangle\mathbf{P}_{N-1}(x_1)]. \tag{49}$$

It is found from (42) and (49) that the structure of singular fields near the crack tip in a piezoelectric bimaterial system is the same as that in a traditional anisotropic bimaterials. However, this point was overlooked in an earlier work (Gao and Wang, 2000).

4. Solution of infinite collinear periodic cracks

For the case of infinite number of collinear periodical interface cracks with equal length $2a$, as shown in Fig. 2, the location of crack tips is specified by

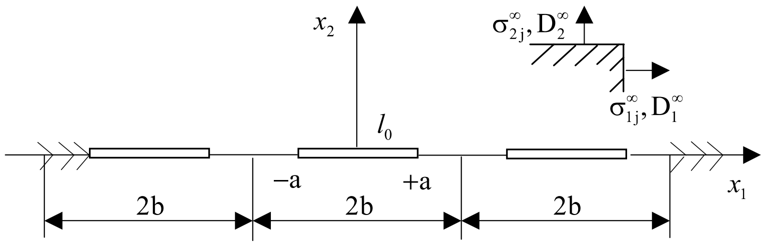

Fig. 2. Periodic interfacial cracks in piezoelectric bimaterials.

$$a_n = 2nb - a, \quad b_n = 2nb + a \quad (n = -\infty, \ldots, 0, \ldots, +\infty). \tag{50}$$

Inserting (50) and (45) into (46) results in

$$X_\alpha(z) = \prod_{n=-\infty}^{+\infty} [(z - 2nb) + a]^{-1/2-\mathrm{i}\varepsilon_\alpha}[(z - 2nb) - a]^{-1/2+\mathrm{i}\varepsilon_\alpha}, \tag{51}$$

$$X_\alpha^{-1}(\infty) = \prod_{n=-\infty}^{+\infty} [z - 2nb + 2\mathrm{i}a\varepsilon_\alpha], \quad \mathbf{P}_{N-1}(z) = \mathbf{0}. \tag{52}$$

Using the identity

$$\sin\pi t = \pi t \prod_{n=1}^{\infty}\left(1 - \frac{t^2}{n^2}\right),$$

it can be shown that (Boniface and Banks-Sills, 2002)

$$X_\alpha^{-1}(\infty)X_\alpha(z) = \frac{\sin\left[\frac{\pi(z+2\mathrm{i}a\varepsilon_\alpha)}{2b}\right]}{\sqrt{\sin\left[\frac{\pi(z+a)}{2b}\right]\sin\left[\frac{\pi(z-a)}{2b}\right]}}\left[\frac{\sin\left[\frac{\pi(z-a)}{2b}\right]}{\sin\left[\frac{\pi(z+a)}{2b}\right]}\right]^{\mathrm{i}\varepsilon_\alpha}. \tag{53}$$

Note

$$\sin\left[\frac{\pi(z \pm a)}{2b}\right] = \sin\frac{\pi z}{2b}\cos\frac{\pi a}{2b} \pm \cos\frac{\pi z}{2b}\sin\frac{\pi a}{2b}.$$

Then, one can rewrite (53) as

$$X_\alpha^{-1}(\infty)X_\alpha(z) = \frac{\sin\left[\frac{\pi}{2b}(z + 2\mathrm{i}a\varepsilon_\alpha)\right]\sec\frac{\pi a}{2b}\sec\frac{\pi z}{2b}}{\sqrt{tg^2\frac{\pi z}{2b} - \mathrm{tg}^2\frac{\pi a}{2b}}}\left[\frac{\mathrm{tg}\frac{\pi z}{2b} - \mathrm{tg}\frac{\pi a}{2b}}{\mathrm{tg}\frac{\pi z}{2b} + \mathrm{tg}\frac{\pi a}{2b}}\right]^{\mathrm{i}\varepsilon_\alpha}. \tag{54}$$

Substituting (51) and (52) into (43) can give the complex potential as

$$\mathbf{K}_3(z) = -\mathbf{Q}\left\langle\left\langle\frac{1 - X_\alpha(z)X_\alpha^{-1}(\infty)}{1 + \mathrm{e}^{2\pi\varepsilon_\alpha}}\right\rangle\right\rangle\mathbf{Q}^{-1}\mathbf{\Lambda}_\sigma^{-1}\sigma_2^\infty. \tag{55}$$

Correspondingly, the stress intensity factor is

$$\mathbf{k}_\sigma = \sqrt{\pi a}[\mathbf{\Lambda}_\sigma\mathbf{Q}]\langle\langle(2a)^{-\mathrm{i}\varepsilon_\alpha}\omega_\alpha\rangle\rangle[\mathbf{\Lambda}_\sigma\mathbf{Q}]^{-1}\boldsymbol{\sigma}_2^\infty, \tag{56}$$

where

$$\omega_\alpha = \sec\frac{\pi a}{2b}\sin\left[\frac{\pi a}{2b}(1+2i\varepsilon_\alpha)\right]\left(\frac{\pi a}{2b}\right)^{i\varepsilon_\alpha - 1/2}\left(\mathrm{tg}\frac{\pi a}{2b}\right)^{-i\varepsilon_\alpha - 1/2}. \tag{57}$$

5. Numerical example: for the case of a single crack

As an example, consider a crack located in $[-a, +a]$, as shown in Fig. 3. In this case, we have

$$X_\alpha(z) = \frac{1}{\sqrt{z^2 - a^2}}\left(\frac{z-a}{z+a}\right)^{i\varepsilon_\alpha}, \quad X_\alpha^{-1}(\infty) = z + 2ia\varepsilon_\alpha, \quad \mathbf{P}_0(z) = \mathbf{0}. \tag{58}$$

The complex potential can be determined from (43) such that

$$\mathbf{K}_3(z) = -\mathbf{Q}\left\langle\left\langle \frac{1 - X_\alpha(z)(z + 2ia\varepsilon_\alpha)}{1 + e^{2\pi\varepsilon_\alpha}} \right\rangle\right\rangle \mathbf{Q}^{-1}\Lambda_\sigma^{-1}\sigma_2^\infty. \tag{59}$$

Substituting (58) into (49) yields the stress intensity factor vector as

$$\mathbf{k}_\sigma = \sqrt{\pi a}[\boldsymbol{\Lambda}_\sigma \mathbf{Q}]\langle\langle (2a)^{-i\varepsilon_\alpha}(1 + 2i\varepsilon_\alpha)\rangle\rangle [\boldsymbol{\Lambda}_\sigma \mathbf{Q}]^{-1}\boldsymbol{\sigma}_2^\infty. \tag{60}$$

On the other hand, if $b \to \infty$ in (57), one has

$$\begin{aligned}\omega_\alpha &= 1 \times \frac{\pi a}{2b}(1 + 2i\varepsilon_\alpha)\left(\frac{\pi a}{2b}\right)^{-1/2}\left(\frac{\pi a}{2b}\right)^{-1/2}, \\ &= 1 + 2i\varepsilon_\alpha.\end{aligned} \tag{61}$$

Inserting (61) into (56) one can reach (60) again.

To obtain the crack opening $\Delta\mathbf{u}^{\mathrm{m}}(x_1)$, we have from (23) and (25) that

$$\Delta\mathbf{u}^{\mathrm{m}}(x_1) = \mathbf{K}_3^+(x_1) - \mathbf{K}_3^-(x_1), \tag{62}$$

and then inserting (59) into (62) and using $X_\alpha^- = -e^{-2\pi\varepsilon_\alpha}X_\alpha^+$ on crack faces, we obtain

$$i\Delta\mathbf{u}^{\mathrm{m}}_{,1}(x_1) = \mathbf{Q}\langle\langle e^{-2\pi\varepsilon_\alpha}X_\alpha^+(x_1)(x_1 + 2ia\varepsilon_\alpha)\rangle\rangle \mathbf{Q}^{-1}\Lambda_\sigma^{-1}\boldsymbol{\sigma}_2^\infty, \quad -a < x_1 < a, \tag{63}$$

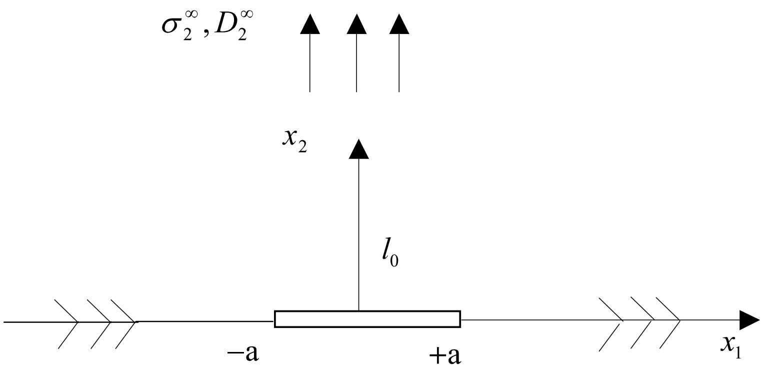

Fig. 3. An interfacial crack in piezoelectric bimaterials.

namely

$$\Delta\mathbf{u}^{\mathrm{m}}_{,1}(x_1) = \mathrm{Im}\left[\mathbf{Q}\langle\langle e^{-2\pi\varepsilon_\alpha} X^+_\alpha(x_1)(x_1 + 2\mathrm{i}a\varepsilon_\alpha)\rangle\rangle\mathbf{Q}^{-1}\Lambda_\sigma^{-1}\right]\boldsymbol{\sigma}_2^\infty, \quad -a < x_1 < a. \tag{64}$$

Taking the integration of (64) with respect to x_1 and noting $\Delta\mathbf{u}^{\mathrm{m}}(a) = 0$, one can obtain (Ting, 1996)

$$\Delta\mathbf{u}^{\mathrm{m}}(x_1) = \mathrm{Im}\left[\mathbf{Q}\langle\langle e^{-2\pi\varepsilon_\alpha}\rangle\rangle\chi^+_\alpha(x_1)\mathbf{Q}^{-1}\Lambda_\sigma^{-1}\right]\boldsymbol{\sigma}_2^\infty, \quad -a < x_1 < a, \tag{65}$$

where

$$\chi^+_\alpha(x_1) = \mathrm{i}\sqrt{a^2 - x_1^2}\, e^{-\pi\varepsilon_\alpha} e^{\mathrm{i}\varepsilon_\alpha X}, \quad X = \ln\left|\frac{x_1 - a}{x_1 + a}\right|, \quad -a < x_1 < a.$$

Eq. (65) can be further reduced to

$$\Delta\mathbf{u}^m(x_1) = \sqrt{a^2 - x_1^2}\,\mathrm{Re}\left[\mathbf{Q}\langle\langle e^{\mathrm{i}\varepsilon_\alpha X}\rangle\rangle\langle\langle e^{-\pi\varepsilon_\alpha}\rangle\rangle\mathbf{Q}^{-1}\Lambda_\sigma^{-1}\right]\boldsymbol{\sigma}_2^\infty, \quad -a < x_1 < a. \tag{66}$$

which shows that the displacement is oscillatory near the crack tip.

On the other hand, substituting (59) into (36) results in the expression of the electric displacement inside the crack as

$$D_2^0(x_1) = D_2^\infty + D_{2\sigma}^0(x_1), \tag{67}$$

where $D_{2\sigma}^0(x_1)$ is the electric displacement induced by the applied mechanical loading, and it is a function of position along the crack line such that

$$\begin{aligned} D_{2\sigma}^0(x_1) = &-[\overline{\boldsymbol{\Lambda}}_{13}\overline{\boldsymbol{\Lambda}}_\sigma^{-1}]\sigma_2^\infty - [\boldsymbol{\Lambda}_{13}\boldsymbol{\Lambda}_\sigma^{-1} - \overline{\boldsymbol{\Lambda}}_{13}\overline{\boldsymbol{\Lambda}}_\sigma^{-1}]\mathbf{Q}\left\langle\left\langle\frac{1}{1 + e^{2\pi\varepsilon_\alpha}}\right\rangle\right\rangle\mathbf{Q}^{-1}\boldsymbol{\Lambda}_\sigma^{-1}\boldsymbol{\sigma}_2^\infty \\ &+ [\boldsymbol{\Lambda}_{13}\boldsymbol{\Lambda}_\sigma^{-1} - \overline{\Lambda}_{13}\overline{\boldsymbol{\Lambda}}_\sigma^{-1}]\mathbf{Q}\left\langle\left\langle\frac{X^+_\alpha(x_1)(x_1 + 2\mathrm{i}a\varepsilon_\alpha)}{1 + e^{2\pi\varepsilon_\alpha}}\right\rangle\right\rangle\mathbf{Q}^{-1}\boldsymbol{\Lambda}_\sigma^{-1}\boldsymbol{\sigma}_2^\infty. \end{aligned} \tag{68}$$

It can be found from (67) and (68) that the applied electric loading induces a constant electric field inside the crack, while the applied mechanical loading produces a singular and oscillatory electric field near the crack tip. This means that the singularity of electric fields near the crack tip is only characterized by the applied mechanical loading.

In general, the material constants involved in (3) have the orders in magnitude as follows:

$$c_{ijkl} = c_{ijkl}^0 \times 10^{10}\ \mathrm{N/m^2}, \quad e_{kij} = e_{kij}^0 \times 10^0\ \mathrm{C/m^2}, \quad \varepsilon_{ij} = \varepsilon_{ij}^0 \times 10^{-10}\ \mathrm{C/Vm}, \tag{69}$$

where c_{ijkl}^0, e_{kij}^0 and ε_{ij}^0 are dimensionless constants which are about the same in order of magnitude.

Substituting (69) into (3) gives

$$\sigma_{ij} = c_{ijkl}^0\gamma_{kl} \times 10^{10}\ \mathrm{N/m^2} - e_{kij}^0 E_k \times 10^0\ \mathrm{C/m^2}, \quad D_k = e_{kij}^0\gamma_{ij} \times 10^0\ \mathrm{C/m^2} + \varepsilon_{kl}^0 E_l \times 10^{-10}\ \mathrm{C/Vm} \tag{70}$$

Let

$$\gamma_{kl} \times 10^{10} = \gamma_{kl}^0, \quad E_k \times 10^0\ \mathrm{C/m^2} = E_k^0\ \mathrm{N/m^2}, \quad \sigma_{ij} = \sigma_{ij}^0\ \mathrm{N/m^2}, \quad D_k = D_k^0 \times 10^{-10}\ \mathrm{C/m^2}, \tag{71}$$

Then, (70) becomes

$$\sigma_{ij}^0 = c_{ijkl}^0\gamma_{kl}^0 - e_{kij}^0 E_k^0, \quad D_k^0 = e_{kij}^0\gamma_{ij}^0 + \varepsilon_{kl}^0 E_l^0. \tag{72}$$

In the following numerical calculation, we consider a transversely isotropic piezoelectric material with the poling direction being parallel to the x_2-axis. In this case, according to the constitutive question (72), the elements in the three matrices in (6) can be inputted by c_{ij}^0, e_{ij}^0 and ε_{ij}^0, respectively (here we used Voigt's notation), and moreover these matrixes can be explicitly expressed as

$$\mathbf{W}^0 = \begin{bmatrix} c_{11}^0 & 0 & 0 & 0 \\ 0 & c_{44}^0 & 0 & e_{15}^0 \\ 0 & 0 & \frac{c_{11}^0 - c_{12}^0}{2} & 0 \\ 0 & e_{15}^0 & 0 & -\varepsilon_{11}^0 \end{bmatrix}, \quad \mathbf{R}^0 = \begin{bmatrix} 0 & c_{13}^0 & 0 & e_{31}^0 \\ c_{44}^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ e_{15}^0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{T}^0 = \begin{bmatrix} c_{44}^0 & 0 & 0 & 0 \\ 0 & c_{33}^0 & 0 & e_{33}^0 \\ 0 & 0 & c_{44}^0 & 0 \\ 0 & e_{33}^0 & 0 & -\varepsilon_{33}^0 \end{bmatrix}. \tag{73}$$

Based on (73), one can obtain highly exact numerical results, since the elements involved in these matrices have about the same orders of magnitude. However, it should be noted that the obtained strain (or displacement) and electric displacement are γ^0 and D^0, while the physical γ and D are given by the replacements of (71).

Below we use the piezoelectric ceramics PZT-4 (located on the upper half-space) and PZT-5H (located on the lower upper half-space) as the model materials for our numerical calculation. The uniform far-fields $\sigma_{22}^{0\infty}$ and $D_2^{0\infty}$ are applied. The properties of PZT-4 are

$$c_{11}^0 = 13.90, \quad c_{12}^0 = 7.78, \quad c_{13}^0 = 7.74,$$

$$c_{33}^0 = 11.30, \quad c_{44}^0 = 2.56,$$

$$e_{31}^0 = -6.98, \quad e_{33}^0 = 13.84, \quad e_{15}^0 = 13.44,$$

$$\varepsilon_{11}^0 = 60.00, \quad \varepsilon_{33}^0 = 54.70$$

and the properties of PZT-5H are

$$c_{11}^0 = 12.60, \quad c_{12}^0 = 5.50, \quad c_{13} = 5.30,$$

$$c_{33}^0 = 11.70, \quad c_{44}^0 = 3.53,$$

$$e_{31}^0 = -6.50, \quad e_{33}^0 = 23.30, \quad e_{15}^0 = 17.00,$$

$$\varepsilon_{11}^0 = 151.00, \quad \varepsilon_{33}^0 = 130.00.$$

Using commercial software Matlab, we have from (68) that

$$D_{2\sigma}^0(x_1) = -2.44\sigma_{22}^{0\infty} + \frac{0.293}{\sqrt{1 - x_*^2}} \mathrm{Im}\left[(x_* + 2\mathrm{i}\varepsilon)\varpi(x_*)\right]\sigma_{22}^{0\infty}, \tag{74}$$

where

$$x_* = \frac{x_1}{a}, \quad (|x_1| < a), \quad \varepsilon \approx 0.02, \quad \varpi(x_*) = \left(\frac{1 - x_*}{1 + x_*}\right)^{\mathrm{i}\varepsilon}.$$

For the special case of a homogeneous material (PZT-4), the electric field inside the crack is described by

$$D_2^0 = D_2^{0\infty} - 2.264\sigma_{22}^{0\infty}. \tag{75}$$

The variation of the electric displacement $D_{2\sigma}^0(x_1)$ along the crack line is plotted in Fig. 4 where the data points are taken within a range $x_1/a = 0, \ldots, 0.999$. In the range, no oscillatory behaviour is visible. As indicated by the analytical result (68), the electric field $E_{2\sigma}^0 = D_{2\sigma}^0/\varepsilon_0$ induced by the applied mechanical loading inside the crack is singular near the tip of an interface crack. However, for a crack in a homogeneous material, Eq. (75) shows that E_2^0 is a constant. In general, the electric field inside the crack consists of two parts: one is E_{2e}^0 induced by the applied electric field, and the other $E_{2\sigma}^0$ induced by the applied mechanical loading. Since the sign of $E_{2\sigma}^0$ is opposite to that of E_{2e}^0, the sum electric field is equal to $E_2^0 = E_{2e}^0 - E_{2\sigma}^0$. This means that a proper ratio of the applied electric loading to the applied mechanical

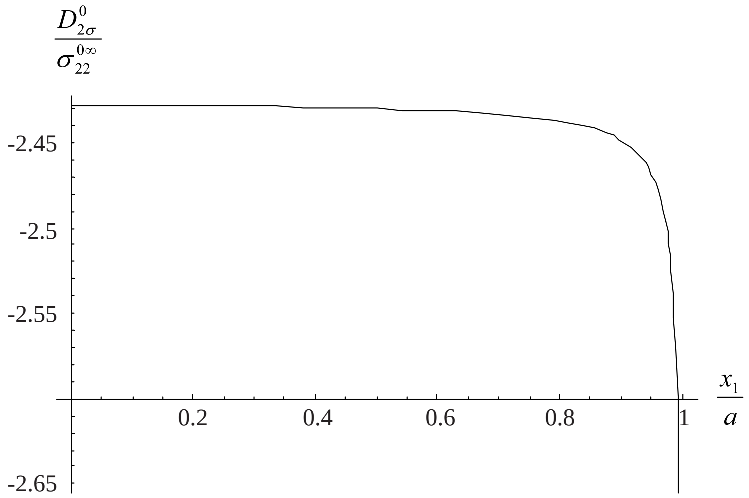

Fig. 4. The electric displacement induced by the mechanical load $\sigma_{22}^{0\infty}$.

loading may lead to a zero electric field inside the crack. For a homogeneous material with a crack, we can show that the ratio is

$$\frac{D_2^{\infty}}{\sigma_{22}^{\infty}} = \frac{H_{42}}{-H_{44}}. \tag{76}$$

In fact, (76) gives the loading condition under which the commonly used impermeable crack model is valid.

6. Conclusions

This work considers the problem of permeable interface cracks in piezoelectric bimaterials based on the Stroh formalism. The considered problem is reduced into an equivalent interface crack problem in purely elastic anisotropic media. This makes it easy to extend the available results concerning interface cracks in traditional anisotropic materials to the corresponding cases of piezoelectric media. The solution of periodic collinear crack is at the first time derived in explicit and closed form. It is found that the electric field inside the cracks is very complex and in general singular near the crack tip. Under such high local electric field, it is believed that partial discharge may occur, accompanying with the formation of initiation electrical tree channels (Dissado and Fothergill, 1992). This process may lead partial material near the crack tip to be electrically broken down (Zhang and Gao, 2003), and therefore the electrical non-linearity will dominate to the fracture behaviour of piezoelectric materials. Thus, it is necessary to develop an electrical non-linearity model for the simulation of interface fracture of piezoelectric materials. This is left to future work.

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